6.1 a) $\begin{pmatrix} 8 & -3 & 1 \\ 5 & 8 & 5 \end{pmatrix}$

b) $\begin{pmatrix} -7 & 20 & 7 \\ 3 & -6 & -9 \end{pmatrix}$

6.2 a) $\begin{pmatrix} 13 & 34 & 18 \\ 8 & 32 & 17 \\ 1 & 5 & 3 \end{pmatrix}$

b) $\begin{pmatrix} -21 & 10 & 12 \\ -4 & -9 & -6 \\ -16 & -20 & -3 \end{pmatrix}$

c) $\begin{pmatrix} -13 & 19 & 15 \\ -21 & 10 & 12 \\ -2 & 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 8 & 32 & 17 \\ -5 & -3 & -1 \\ -12 & -13 & -8 \end{pmatrix}$

e) $\begin{pmatrix} 13 & 18 \\ 3 & 5 \end{pmatrix}$

f) $\begin{pmatrix} 4 & 10 & 12 & 29 \\ 1 & 4 & 3 & 8 \\ 0 & -4 & 0 & -2 \\ 1 & 8 & 3 & 10 \end{pmatrix}$

6.3 a) eine Nullspalte

b) Spalte 3 Vielfaches von Spalte 1

c) zwei gleiche Zeilen

d) Zeile 2 plus Zeile 4 ist Vielfaches von Zeile 3

6.4 a)

$$\begin{array}{c} \qquad\qquad -7S_1 \\ \begin{vmatrix} -2 & 8 & 2 \\ 1 & 0 & 7 \\ 4 & 3 & 1 \end{vmatrix} \end{array} = \begin{vmatrix} -2 & 8 & 16 \\ 1 & 0 & 0 \\ 4 & 3 & -27 \end{vmatrix} = -\begin{vmatrix} 8 & 16 \\ 3 & -27 \end{vmatrix} = 264$$

b)

$$\begin{array}{c} \qquad\qquad -4S_2 \\ \begin{vmatrix} 3 & 4 & -10 \\ -7 & 4 & 1 \\ 0 & 2 & 8 \end{vmatrix} \end{array} = \begin{vmatrix} 3 & 4 & -26 \\ -7 & 4 & -15 \\ 0 & 2 & 0 \end{vmatrix} = -2\begin{vmatrix} 3 & -26 \\ -7 & -15 \end{vmatrix} = 454$$

c)

$$\begin{vmatrix} 2 & 5 & 1 & 4 \\ -5 & 3 & 0 & 0 \\ 1 & 7 & 0 & -3 \\ 9 & 3 & 4 & 5 \end{vmatrix}\begin{matrix} \\ \\ \\ -4Z_1 \end{matrix} = \begin{vmatrix} 2 & 5 & 1 & 4 \\ -5 & 3 & 0 & 0 \\ 1 & 7 & 0 & -3 \\ 1 & -17 & 0 & -11 \end{vmatrix}$$

$$= \begin{vmatrix} -5 & 3 & 0 \\ 1 & 7 & -3 \\ 1 & -17 & -11 \end{vmatrix}\begin{matrix} -5 & 3 \\ 1 & 7 \\ 1 & -17 \end{matrix} = 5\cdot 7\cdot 11 - 3\cdot 3 + 17\cdot 3\cdot 5 + 11\cdot 3$$

$$= 664$$

d)

$$\begin{vmatrix} 1 & 0 & -1 & 2 \\ 2 & 3 & 2 & -2 \\ 2 & 4 & 2 & 1 \\ 3 & 1 & 5 & -3 \end{vmatrix}\begin{matrix} \\ -3Z_4 \\ -4Z_4 \\ \\ \end{matrix} = \begin{vmatrix} 1 & 0 & -1 & 2 \\ -7 & 0 & -13 & 7 \\ -10 & 0 & -18 & 13 \\ 3 & 1 & 5 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ -7 & -13 & 7 \\ -10 & -18 & 13 \end{vmatrix}\begin{matrix} \\ +7Z_1 \\ +10Z_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & -1 & 2 \\ 0 & -20 & 21 \\ 0 & -28 & 33 \end{vmatrix} = \begin{vmatrix} -20 & 21 \\ -28 & 33 \end{vmatrix} = -72$$

6.5 a)

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ -3Z_1 \\ -Z_1 \end{matrix} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -5 & -6 & -3 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 1 \end{array}\right)\begin{matrix} \\ \cdot(-1) \\ \cdot(-1) \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 5 & 6 & 3 & -1 & 0 \end{array}\right)\begin{matrix} \\ \\ -5Z_2 \end{matrix} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & -1 & 5 \end{array}\right)\begin{matrix} -2Z_3 \\ -Z_3 \\ \\ \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 5 & 2 & -10 \\ 0 & 1 & 0 & 3 & 1 & -6 \\ 0 & 0 & 1 & -2 & -1 & 5 \end{array}\right)\begin{matrix} -2Z_3 \\ \\ \\ \end{matrix} \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right.\hspace{-1em}\underbrace{\left.\begin{array}{ccc} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{array}\right)}_{A^{-1}}$$

b)

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right)\begin{matrix} \\ -Z_1 \\ -Z_1 \end{matrix} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)\begin{matrix} \\ \\ +Z_2 \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 1 & 0 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{array}\right)\begin{matrix} \\ \cdot(-1) \\ \cdot(-1) \end{matrix} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)\begin{matrix} -Z_3 \\ -2Z_3 \\ \\ \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & -3 & 1 & 2 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right)\begin{matrix} -Z_2 \\ \\ \\ \end{matrix} \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right.\hspace{-1em}\underbrace{\left.\begin{array}{ccc} 2 & 0 & -1 \\ -3 & 1 & 2 \\ 2 & -1 & -1 \end{array}\right)}_{A^{-1}}$$

c)

$$\left(\begin{array}{ccc|ccc} 2 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \\ \cdot(-1) \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & -2 & 1 & 1 & 0 & 0 \end{array}\right)\begin{array}{l} \\ \\ -2z_1 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 2 \end{array}\right)\begin{array}{l} +2z_2 \\ \\ -2z_2 \end{array} \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 2 \end{array}\right)$$

$$\underbrace{\begin{matrix} 0 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & -2 & 2 \end{matrix}}_{A^{-1}}$$

d)

$$\left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 4 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ -z_1 \\ -z_1 \\ -2z_1 \end{array} \longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & -2 & -2 & -1 & -1 & 0 & 1 & 0 \\ 0 & -2 & -3 & -1 & -2 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \\ -2z_2 \\ -2z_2 \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & -3 & 1 & 0 & -2 & 0 & 1 \end{array}\right)\begin{array}{l} \\ \cdot(-1) \\ \cdot(-3) \\ \cdot(-2) \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 6 & -3 & -3 & 6 & -3 & 0 \\ 0 & 0 & 6 & -2 & 0 & 4 & 0 & -2 \end{array}\right)\begin{array}{l} \\ \\ \\ -z_3 \end{array} \longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 6 & -3 & -3 & 6 & -3 & 0 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)\begin{array}{l} \\ \\ \cdot\frac{1}{3} \\ \\ \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)\begin{array}{l} -2z_4 \\ -z_4 \\ +z_4 \\ \\ \end{array} \longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 0 & -5 & 4 & -6 & 4 \\ 0 & 1 & 0 & 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 2 & 0 & 2 & 0 & 2 & -2 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)\begin{array}{l} \\ \\ \cdot\frac{1}{2} \\ \\ \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 3 & 0 & -5 & 4 & -6 & 4 \\ 0 & 1 & 0 & 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)\begin{array}{l} -3z_3 \\ \\ \\ \\ \end{array} \longrightarrow \left(\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & -8 & 4 & -9 & 7 \\ 0 & 1 & 0 & 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)\begin{array}{l} -3z_2 \\ \\ \\ \\ \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2 & 1 & -3 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 3 & -2 & 3 & -2 \end{array}\right)$$

$$\underbrace{\begin{matrix} -2 & 1 & 0 & 1 \\ -2 & 1 & -3 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & -2 & 3 & -2 \end{matrix}}_{A^{-1}}$$

6.6

$$A=\begin{pmatrix}2&-2&1\\0&1&0\\-1&2&0\end{pmatrix}\quad A^2=\begin{pmatrix}3&-4&2\\0&1&0\\-2&4&-1\end{pmatrix}\quad 2A=\begin{pmatrix}4&-4&2\\0&2&0\\-2&4&0\end{pmatrix}$$

$$A^2-2A=\begin{pmatrix}3&-4&2\\0&1&0\\-2&4&-1\end{pmatrix}-\begin{pmatrix}4&-4&2\\0&2&0\\-2&4&0\end{pmatrix}=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}=-E$$

$$A^2-2A+E=0 \qquad 2A-A^2=E \qquad A\underbrace{(2E-A)}_{A^{-1}}=E$$

$$\Rightarrow A^{-1}=2E-A$$

$$A^{-1}=\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}-\begin{pmatrix}2&-2&1\\0&1&0\\-1&2&0\end{pmatrix}=\begin{pmatrix}0&2&-1\\0&1&0\\1&-2&2\end{pmatrix}$$

6.7

$$X^2-2X+E=A^2 \qquad (X-E)^2=A^2 \qquad X-E=\pm A$$

$$X=E\pm A$$

$$X_1=E+A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}+\begin{pmatrix}1&2&1\\1&0&1\\2&0&2\end{pmatrix}=\begin{pmatrix}2&2&1\\1&1&1\\2&0&3\end{pmatrix}$$

$$X_2=E-A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-\begin{pmatrix}1&2&1\\1&0&1\\2&0&2\end{pmatrix}=\begin{pmatrix}0&-2&-1\\-1&1&-1\\-2&0&-1\end{pmatrix}$$

6.8

Vollständige Induktion

$$n=1 \qquad \begin{pmatrix}1&2\\0&2\end{pmatrix}=\begin{pmatrix}1&2^2-2\\0&2\end{pmatrix}=\begin{pmatrix}1&2\\0&2\end{pmatrix} \quad \checkmark$$

$$\begin{pmatrix}1&2\\0&2\end{pmatrix}^n=\begin{pmatrix}1&2^{n+1}-2\\0&2^n\end{pmatrix} \Rightarrow \begin{pmatrix}1&2\\0&2\end{pmatrix}^n\begin{pmatrix}1&2\\0&2\end{pmatrix}=\begin{pmatrix}1&2^{n+1}-2\\0&2^n\end{pmatrix}\begin{pmatrix}1&2\\0&2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1&2\\0&2\end{pmatrix}^{n+1}=\begin{pmatrix}1&2+(2^{n+1}-2)2\\0&2^n\cdot 2\end{pmatrix}=\begin{pmatrix}1&2^{n+2}-2\\0&2^{n+1}\end{pmatrix} \quad \checkmark$$

6.9 a)

$$\left(\begin{array}{ccc|c}1&2&-3&-1\\3&-1&2&7\\5&3&-4&2\end{array}\right)\begin{array}{l}\\-3z_1\\-5z_1\end{array} \longrightarrow \left(\begin{array}{ccc|c}1&2&-3&-1\\0&-7&11&10\\0&-7&11&7\end{array}\right)\begin{array}{l}\\\\-z_2\end{array} \longrightarrow \left(\begin{array}{ccc|c}1&2&-3&-1\\0&-7&11&10\\0&0&0&-3\end{array}\right)$$

nicht lösbar

b)

$$\left(\begin{array}{ccc|c}2&1&-2&10\\3&2&2&1\\5&4&3&4\end{array}\right)\begin{array}{l}\\-\frac{3}{2}z_1\\-\frac{5}{2}z_1\end{array} \longrightarrow \left(\begin{array}{ccc|c}2&1&-2&10\\0&\frac{1}{2}&5&-14\\0&\frac{3}{2}&8&-21\end{array}\right)\begin{array}{l}\\\cdot 2\\\cdot 2\end{array} \longrightarrow \left(\begin{array}{ccc|c}2&1&-2&10\\0&1&10&-28\\0&3&16&-42\end{array}\right)\begin{array}{l}\\\\-3z_2\end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c}2&1&-2&10\\0&1&10&-28\\0&0&-14&42\end{array}\right)$$

$$\begin{aligned}2x_1+x_2-2x_3&=10 &&\Rightarrow x_1=1\\ x_2+10x_3&=-28 &&\Rightarrow x_2=2\\ -14x_3&=42 &&\Rightarrow x_3=-3\end{aligned}$$

c)

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 2 & -1 & 4 & 2 \\ 4 & 3 & -2 & 14 \end{array}\right)\begin{array}{l} \\ -2z_1 \\ -4z_1 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & -5 & 10 & -10 \end{array}\right)\begin{array}{l} \\ \\ -z_2 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & -5 & 10 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right)\begin{array}{l} \\ \cdot\left(-\frac{1}{5}\right) \\ \\ \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 6 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

unendlich viele Lösungen $\quad x_3 = t$

$$x_1 + 2x_2 - 3x_3 = 6 \quad \Rightarrow \quad x_1 = 2 - t$$
$$x_2 - 2x_3 = 2 \quad \Rightarrow \quad x_2 = 2 + 2t$$

d)

$$\left(\begin{array}{cccc|c} 3 & -1 & -2 & 1 & 1 \\ 2 & 1 & 1 & 3 & 6 \\ -1 & 3 & 2 & 4 & 1 \\ -2 & -2 & 3 & -2 & 7 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 2 & 1 & 1 & 3 & 6 \\ 3 & -1 & -2 & 1 & 1 \\ -2 & -2 & 3 & -2 & 7 \end{array}\right)\begin{array}{l} \\ +2z_1 \\ +3z_1 \\ -2z_1 \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 0 & 7 & 5 & 11 & 8 \\ 0 & 8 & 4 & 13 & 4 \\ 0 & -8 & -1 & -10 & 5 \end{array}\right)\begin{array}{l} \\ +z_3 \\ \\ +z_3 \end{array} \longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 0 & -1 & 1 & -2 & 4 \\ 0 & 8 & 4 & 13 & 4 \\ 0 & 0 & 3 & 3 & 9 \end{array}\right)\begin{array}{l} \\ \\ +8z_2 \\ \cdot 4 \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 0 & -1 & 1 & -2 & 4 \\ 0 & 0 & 12 & -3 & 36 \\ 0 & 0 & 12 & 12 & 36 \end{array}\right)\begin{array}{l} \\ \\ \\ -z_3 \end{array} \longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 0 & -1 & 1 & -2 & 4 \\ 0 & 0 & 12 & -3 & 36 \\ 0 & 0 & 0 & 15 & 0 \end{array}\right)\begin{array}{l} \\ \\ \cdot\frac{1}{3} \\ \cdot\frac{1}{15} \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|c} -1 & 3 & 2 & 4 & 1 \\ 0 & -1 & 1 & -2 & 4 \\ 0 & 0 & 4 & -1 & 12 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$-x_1 + 3x_2 + 2x_3 + 4x_4 = 1 \quad \Rightarrow \quad x_1 = 2$$
$$-x_2 + x_3 - 2x_4 = 4 \quad \Rightarrow \quad x_2 = -1$$
$$4x_3 - x_4 = 12 \quad \Rightarrow \quad x_3 = 3$$
$$x_4 = 0$$

e)

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 4 \\ 1 & 1 & 2 & -1 & -3 \\ 2 & 2 & 3 & 1 & 1 \\ 3 & 3 & 4 & 3 & 5 \\ 5 & 5 & 7 & 4 & 6 \end{array}\right)\begin{array}{l} \\ -z_1 \\ -2z_1 \\ -3z_1 \\ -5z_1 \end{array} \longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 4 \\ 0 & 0 & 1 & -3 & -7 \\ 0 & 0 & 1 & -3 & -7 \\ 0 & 0 & 1 & -3 & -7 \\ 0 & 0 & 2 & -6 & -14 \end{array}\right)\begin{array}{l} \\ \\ -z_2 \\ -z_2 \\ -2z_2 \end{array}$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 4 \\ 0 & 0 & 1 & -3 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 + x_2 + x_3 + 2x_4 = 4$$
$$x_3 - 3x_4 = -7$$

$x_2 = t \quad x_4 = s \quad x_3 = 3s - 7 \quad x_1 = 11 - t - 5s$

6.10

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} \begin{array}{l} -\lambda z_3 \\ -z_3 \\ \end{array} = \begin{vmatrix} 0 & 1-\lambda & 1-\lambda^2 \\ 0 & \lambda-1 & 1-\lambda \\ 1 & 1 & \lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1-\lambda^2 \\ \lambda-1 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)^2 - (\lambda-1)(1-\lambda^2) = (1-\lambda)^2 + (1-\lambda)(1-\lambda^2)$$

$$= (1-\lambda)\left(1-\lambda + 1-\lambda^2\right) = (1-\lambda)(2-\lambda-\lambda^2) = (\lambda-1)(\lambda^2+\lambda-2)$$

$$= (\lambda-1)(\lambda-1)(\lambda+2) = (\lambda-1)^2(\lambda+2) = 0 \quad \text{für } \lambda = 1 \text{ oder } \lambda = -2$$

$\lambda \neq 1 \wedge \lambda = -2 \Rightarrow$ eindeutig lösbar

$\lambda = -2$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 1 & -2 & 1 & 1 \\ -2 & 1 & 1 & 1 \end{array}\right) \begin{array}{l} \\ -z_1 \\ +2z_1 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 3 \end{array}\right) \begin{array}{l} \\ \\ +z_2 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right) \quad \text{nicht lösbar}$$

$\lambda = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right) \begin{array}{l} \\ -z_1 \\ -z_1 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 + x_2 + x_3 = 1 \qquad x_3 = t \quad x_2 = s \qquad x_1 = 1 - s - t$$

$\lambda \neq 1 \wedge \lambda \neq -2$ $\quad \left(\begin{array}{ccc|c} \lambda & 1 & 1 & 1 \\ 1 & \lambda & 1 & 1 \\ 1 & 1 & \lambda & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 1 & \lambda & 1 & 1 \\ \lambda & 1 & 1 & 1 \end{array}\right) \begin{array}{l} \\ -z_1 \\ -\lambda z_1 \end{array}$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & 0 \\ 0 & 1-\lambda & 1-\lambda^2 & 1-\lambda \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & \lambda-1 & 1-\lambda & 0 \\ 0 & (1-\lambda) & (1-\lambda)(1+\lambda) & 1-\lambda \end{array}\right) \begin{array}{l} \\ \cdot \frac{1}{1-\lambda} \\ \cdot \frac{1}{1-\lambda} \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1+\lambda & 1 \end{array}\right) \begin{array}{l} \\ \\ +z_2 \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & \lambda & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2+\lambda & 1 \end{array}\right)$$

(I) $x_1 + x_2 + \lambda x_3 = 1$

(II) $-x_2 + x_3 = 0$

(III) $(2+\lambda)x_3 = 1 \implies x_3 = \frac{1}{2+\lambda} = \frac{1}{\lambda+2}$ in (I), (II)

(II) $\implies$ $x_2 = x_3 = \frac{1}{\lambda+2}$ in (I)

(I) $\implies$ $x_1 = 1 - x_2 - \lambda x_3 = 1 - \frac{1}{\lambda+2} - \frac{\lambda}{\lambda+2} = \frac{\lambda+2}{\lambda+2} - \frac{1}{\lambda+2} - \frac{\lambda}{\lambda+2}$

$$= \frac{\lambda+2-1-\lambda}{\lambda+2} = \frac{1}{\lambda+2}$$

Cramer-Regel

$$\det A = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda-1)^2(\lambda+2)$$

$$\det A_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} \begin{matrix} \\ -z_1 \\ -z_1 \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & \lambda-1 & 0 \\ 0 & 0 & \lambda-1 \end{vmatrix} = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = (\lambda-1)^2$$

$$\det A_2 = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \lambda \end{vmatrix} \begin{matrix} -z_2 \\ \\ -z_2 \end{matrix} = \begin{vmatrix} \lambda-1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & \lambda-1 \end{vmatrix} = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = (\lambda-1)^2$$

$$\det A_3 = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} -z_3 \\ -z_3 \\ \\ \end{matrix} = \begin{vmatrix} \lambda-1 & 0 & 0 \\ 0 & \lambda-1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda-1 \end{vmatrix} = (\lambda-1)^2$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{(\lambda-1)^2}{(\lambda-1)^2(\lambda+2)} = \frac{1}{\lambda+2}$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{(\lambda-1)^2}{(\lambda-1)^2(\lambda+2)} = \frac{1}{\lambda+2}$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{(\lambda-1)^2}{(\lambda-1)^2(\lambda+2)} = \frac{1}{\lambda+2}$$

6.11 a)

$$\begin{vmatrix} 2 & \mu & 1 \\ 3 & 4 & -1 \\ 1 & -\mu & 2 \end{vmatrix} \begin{matrix} \\ +z_1 \\ -2z_1 \end{matrix} = \begin{vmatrix} 2 & \mu & 1 \\ 5 & \mu+4 & 0 \\ -3 & -3\mu & 0 \end{vmatrix} = \begin{vmatrix} 5 & \mu+4 \\ -3 & -3\mu \end{vmatrix}$$

$$= -15\mu + 3(\mu+4) = 12 - 12\mu = 12(1-\mu) = 0 \quad \text{für } \mu = 1$$

$$\mu = 1 \quad \left(\begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 3 & 4 & -1 & 3 \\ 1 & -1 & 2 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 3 & 4 & -1 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right) \begin{matrix} \\ -3z_1 \\ -2z_1 \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 7 & -7 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right) \begin{matrix} \\ \cdot \frac{1}{7} \\ \cdot \frac{1}{3} \end{matrix} \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \begin{matrix} \\ \\ -z_2 \end{matrix}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - x_2 + 2x_3 = 1$

$x_2 - x_3 = 0$

unendlich viele Lösungen

$x_1 = 1 - t$

$x_2 = t$

$x_3 = t$

$$\left(\begin{array}{ccc|c} 2 & \mu & 1 & 2 \\ 3 & 4 & -1 & 3 \\ 1 & -\mu & 2 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & -\mu & 2 & 1 \\ 3 & 4 & -1 & 3 \\ 2 & \mu & 1 & 2 \end{array}\right)\begin{array}{l} \\ -3z_1 \\ -2z_1 \end{array}$$

$$\to \left(\begin{array}{ccc|c} 1 & -\mu & 2 & 1 \\ 0 & 4+3\mu & -7 & 0 \\ 0 & 3\mu & -3 & 0 \end{array}\right)\begin{array}{l} \\ -z_3 \\ \\ \end{array} \longrightarrow \left(\begin{array}{ccc|c} 1 & -\mu & 2 & 1 \\ 0 & 4 & -4 & 0 \\ 0 & 3\mu & -3 & 0 \end{array}\right)\begin{array}{l} \\ \cdot\frac{1}{4} \\ \cdot\frac{1}{3} \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & -\mu & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & \mu & -1 & 0 \end{array}\right)\begin{array}{l} \\ \\ -\mu z_2 \end{array} \to \left(\begin{array}{ccc|c} 1 & -\mu & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & \mu-1 & 0 \end{array}\right)$$

$$\begin{aligned} x_1 - \mu x_2 + 2x_3 &= 1 \\ x_2 - x_3 &= 0 \\ (\mu-1)x_3 &= 0 \end{aligned} \qquad \mu \neq 1 \Rightarrow \qquad \begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$$

Cramer-Regel $\quad \det A = \begin{vmatrix} 2 & \mu & 1 \\ 3 & 4 & -1 \\ 1 & -\mu & 2 \end{vmatrix} = 12(1-\mu)$

$$\det A_1 = \begin{vmatrix} 2 & \mu & 1 \\ 3 & 4 & -1 \\ 1 & -\mu & 2 \end{vmatrix} = \det A$$

$\det A_2 = \begin{vmatrix} 2 & 2 & 1 \\ 3 & 3 & -\mu \\ 1 & 1 & 2 \end{vmatrix} = 0 \quad$ zwei gleiche Spalten

$\det A_3 = \begin{vmatrix} 2 & \mu & 2 \\ 3 & 4 & 3 \\ 1 & -\mu & 1 \end{vmatrix} = 0 \quad$ zwei gleiche Spalten

$$\mu \neq 1 \qquad x_1 = \frac{\det A_1}{\det A} = 1 \qquad x_2 = \frac{\det A_2}{\det A} = 0 \qquad x_3 = \frac{\det A_3}{\det A} = 0$$

b)

$$\vec{x} = \begin{pmatrix} 1-t \\ t \\ t \end{pmatrix} \qquad |\vec{x}| = \sqrt{(1-t)^2 + t^2 + t^2} = \sqrt{2}$$

$$(1-t)^2 + t^2 + t^2 = 1 - 2t + 3t^2 = 2 \qquad 3t^2 - 2t - 1 = 0$$

$t_1 = 1 \quad t_2 = -\frac{1}{3} \quad$ ganzzahlig $\Rightarrow t = t_1 = 1 \qquad \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \vec{x_1}$

c)

$$\vec{x} = \begin{pmatrix} 1-t \\ t \\ t \end{pmatrix} \quad \vec{x_1} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \vec{x} = \sqrt{(1-t)^2 + t^2 + t^2} = \sqrt{3t^2-2t+1}$$

$$|\vec{x_1}| = \sqrt{2}$$

$$\cos(45°) = \frac{\vec{x} \cdot \vec{x_1}}{|\vec{x}||\vec{x_1}|} \qquad \vec{x} \cdot \vec{x_1} = \begin{pmatrix} 1-t \\ t \\ t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2t$$

$$\frac{1}{2}\sqrt{2} = \frac{2t}{\sqrt{3t^2-2t+1}\,\sqrt{2}} \qquad 1 = \frac{2t}{\sqrt{3t^2-2t+1}} \quad (*)$$

$$2t = \sqrt{3t^2-2t+1} \qquad 4t^2 = 3t^2-2t+1$$

$$t^2+2t-1 = 0 \qquad t_1 = \sqrt{2}-1 \qquad t_2 = -\sqrt{2}-1 \quad \text{widerspricht } (*)$$

$$t = t_1 = \sqrt{2}-1$$

$$\vec{x} = \begin{pmatrix} 2-\sqrt{2} \\ \sqrt{2}-1 \\ \sqrt{2}-1 \end{pmatrix}$$

d)

$$A = \begin{pmatrix} 2 & \mu & 1 \\ 3 & 4 & -1 \\ 1 & -\mu & 2 \end{pmatrix} \qquad \det A = \begin{vmatrix} 2 & \mu & 1 \\ 3 & 4 & -1 \\ 1 & -\mu & 2 \end{vmatrix} = 12(1-\mu) = -12(\mu-1)$$

$$b_{11} = (-1)^{1+1} \det U_{11} = \begin{vmatrix} 4 & -1 \\ -\mu & 2 \end{vmatrix} = 8-\mu$$

$$b_{12} = (-1)^{1+2} \det U_{12} = -\begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = -7$$

$$b_{13} = (-1)^{1+3} \det U_{13} = \begin{vmatrix} 3 & 4 \\ 1 & -\mu \end{vmatrix} = -3\mu-4$$

$$b_{21} = (-1)^{2+1} \det U_{21} = -\begin{vmatrix} \mu & 1 \\ -\mu & 2 \end{vmatrix} = -3\mu$$

$$b_{22} = (-1)^{2+2} \det U_{22} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$$b_{23} = (-1)^{2+3} \det U_{23} = -\begin{vmatrix} 2 & \mu \\ 1 & -\mu \end{vmatrix} = 3\mu$$

$$b_{31} = (-1)^{3+1} \det U_{31} = \begin{vmatrix} \mu & 1 \\ 4 & -1 \end{vmatrix} = -\mu-4$$

$$b_{32} = (-1)^{3+2} \det U_{32} = -\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = 5$$

$$b_{33} = (-1)^{3+3} \det U_{33} = \begin{vmatrix} 2 & \mu \\ 3 & 4 \end{vmatrix} = 8-3\mu$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 8-\mu & -7 & -3\mu-4 \\ -3\mu & 3 & 3\mu \\ -\mu-4 & 5 & 8-3\mu \end{pmatrix}$$

$$B^T = \begin{pmatrix} 8-\mu & -3\mu & -\mu-4 \\ -7 & 3 & 5 \\ -3\mu-4 & 3\mu & 8-3\mu \end{pmatrix}$$

$$A^{-1} = \frac{B^T}{\det A} = \frac{1}{-12(\mu-1)} \begin{pmatrix} 8-\mu & -3\mu & -\mu-4 \\ -7 & 3 & 5 \\ -3\mu-4 & 3\mu & 8-3\mu \end{pmatrix} = \begin{pmatrix} \frac{\mu-8}{12(\mu-1)} & \frac{\mu}{4(\mu-1)} & \frac{\mu+4}{12(\mu-1)} \\ \frac{7}{12(\mu-1)} & -\frac{1}{4(\mu-1)} & -\frac{5}{12(\mu-1)} \\ \frac{3\mu+4}{12(\mu-1)} & -\frac{\mu}{4(\mu-1)} & \frac{3\mu-8}{12(\mu-1)} \end{pmatrix}$$

e)

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 4 & -1 \\ 1 & 0 & 2 \end{pmatrix} \qquad \det(A-\lambda E) = \left| \begin{matrix} 2-\lambda & 0 & 1 \\ 3 & 4-\lambda & -1 \\ 1 & 0 & 2-\lambda \end{matrix} \right| \begin{matrix} 2-\lambda & 0 \\ 3 & 4-\lambda \\ 1 & 0 \end{matrix}$$

$$= (2-\lambda)(4-\lambda)(2-\lambda) - (4-\lambda)$$

$$= (4-\lambda)\left((2-\lambda)^2 - 1\right) = 0$$

$$\lambda = 4 \qquad (2-\lambda)^2 = 1 \qquad 2-\lambda = \pm 1 \qquad \lambda = 2 \pm 1$$

$\lambda_1 = 1$

$\lambda_2 = 3$

$\lambda_3 = 4$

$$\lambda = \lambda_1 = 1 \qquad \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix} \begin{matrix} \\ -3z_1 \\ -z_1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & -4 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 + x_3 = 0 \qquad x_1 = -t$$

$$3x_2 - 4x_3 = 0 \qquad x_3 = t \quad x_2 = \frac{4}{3}t$$

$$\vec{x} = \begin{pmatrix} -t \\ \frac{4}{3}t \\ t \end{pmatrix} = \frac{1}{3}t \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix} = t' \begin{pmatrix} -3 \\ 4 \\ 3 \end{pmatrix}$$

$$\lambda = \lambda_2 = 3 \qquad \begin{pmatrix} -1 & 0 & 1 \\ 3 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{matrix} \\ +3z_1 \\ +z_1 \end{matrix} \longrightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$-x_1 + x_3 = 0 \qquad x_1 = t$$

$$x_2 + 2x_3 = 0 \qquad x_3 = t \quad x_2 = -2t$$

$$\vec{x} = \begin{pmatrix} t \\ -2t \\ t \end{pmatrix} = t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\lambda = \lambda_3 = 4$ $\quad \begin{pmatrix} -2 & 0 & 1 \\ 3 & 0 & -1 \\ 1 & 0 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 3 & 0 & -1 \\ -2 & 0 & 1 \end{pmatrix} \begin{matrix} \\ -3Z_1 \\ +2Z_1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$

$$\begin{aligned} x_1 - 2x_3 &= 0 \\ 5x_3 &= 0 \end{aligned} \qquad x_3 = 0 \quad x_1 = 0 \quad x_2 = t$$

$$\vec{x}' = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} = t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

6.12 a)

$$\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = (3-\lambda)(-2-\lambda) + 4 = (\lambda-3)(\lambda+2)+4$$
$$= \lambda^2 - \lambda - 2 = 0 \quad \text{für } \lambda = -1 \text{ oder } \lambda = 2$$

$\lambda = -1$ $\quad \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} \begin{matrix} \\ -2Z_1 \end{matrix} \longrightarrow \begin{pmatrix} 4 & -2 \\ 0 & 0 \end{pmatrix}$

$$4x_1 - 2x_2 = 0 \qquad x_2 = t \qquad x_1 = \frac{t}{2}$$

$$\vec{x} = \begin{pmatrix} \frac{t}{2} \\ t \end{pmatrix} = \frac{t}{2} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = t' \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\lambda = 2$ $\quad \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix} \begin{matrix} \\ -2Z_1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix} \qquad \begin{aligned} &x_1 - 2x_2 = 0 \\ &x_2 = t \quad x_1 = 2t \end{aligned}$

$$\vec{x} = \begin{pmatrix} 2t \\ t \end{pmatrix} = t \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

b)

$$\begin{vmatrix} 1-\lambda & -1 & -1 \\ 1 & 3-\lambda & 1 \\ -3 & 1 & -1-\lambda \end{vmatrix} \begin{matrix} \\ +Z_1 \\ \\ \end{matrix} = \begin{array}{c} -S_2 \\ \begin{vmatrix} 1-\lambda & -1 & -1 \\ 2-\lambda & 2-\lambda & 0 \\ -3 & 1 & -1-\lambda \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} 2-\lambda & -1 & -1 \\ 0 & 2-\lambda & 0 \\ -4 & -1 & -1-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} 2-\lambda & -1 \\ -4 & -1-\lambda \end{vmatrix} = (2-\lambda)\left[(2-\lambda)(-1-\lambda) - 4\right]$$

$$= (2-\lambda)(\lambda^2 - \lambda - 6) = 0 \quad \text{für } \lambda = 2 \text{ oder } \lambda = -2 \text{ oder } \lambda = 3$$

$$\lambda=-2 \quad \begin{pmatrix} 3 & -1 & -1 \\ 1 & 5 & 1 \\ -3 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 3 & -1 & -1 \\ -3 & 1 & 1 \end{pmatrix} \begin{matrix} \\ \\ +z_2 \end{matrix} \rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 3 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} \\ -3z_1 \\ \\ \end{matrix}$$

$$\rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 0 & -16 & -4 \\ 0 & 0 & 0 \end{pmatrix} \cdot \left(-\frac{1}{4}\right) \rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 + 5x_2 + x_3 &= 0 \\ 4x_2 + x_3 &= 0 \qquad x_3 = t \Rightarrow x_2 = -\frac{1}{4}t \Rightarrow x_1 = \frac{1}{4}t \end{aligned}$$

$$\vec{x} = \begin{pmatrix} \frac{1}{4}t \\ -\frac{1}{4}t \\ t \end{pmatrix} = \frac{1}{4}t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = t' \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

$$\lambda=2 \quad \begin{pmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ -3 & 1 & -3 \end{pmatrix} \begin{matrix} \cdot(-1) \\ \\ \\ \end{matrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ -3 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \\ +3z_1 \\ -z_1 \end{matrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \qquad x_3 = t \quad x_1 = -t \\ 4x_2 \quad &= 0 \end{aligned}$$

$$\vec{x} = \begin{pmatrix} -t \\ 0 \\ t \end{pmatrix} = t \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda=3 \quad \begin{pmatrix} -2 & -1 & -1 \\ 1 & 0 & 1 \\ -3 & 1 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ -2 & -1 & -1 \\ -3 & 1 & -4 \end{pmatrix} \begin{matrix} \\ +2z_1 \\ +3z_1 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} \\ \\ +z_2 \end{matrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} x_1 \quad + x_3 &= 0 \\ -x_2 + x_3 &= 0 \end{aligned} \qquad \begin{aligned} x_3 &= t \\ x_2 &= t \\ x_1 &= -t \end{aligned}$$

$$\vec{x} = \begin{pmatrix} -t \\ t \\ t \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

c)

$$\begin{array}{c} \\ \end{array} \begin{vmatrix} 3-\lambda & 1 & -1 \\ 1 & 3-\lambda & -1 \\ 3 & 3 & -1-\lambda \end{vmatrix} \overset{+S_2}{=} \begin{vmatrix} 3-\lambda & 1 & 0 \\ 1 & 3-\lambda & 2-\lambda \\ 3 & 3 & 2-\lambda \end{vmatrix} \begin{matrix} \\ -z_3 \\ \\ \end{matrix}$$

$$= \begin{vmatrix} 3-\lambda & 1 & 0 \\ -2 & -\lambda & 0 \\ 3 & 3 & 2-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix}$$

$$= (2-\lambda)(\lambda^2 - 3\lambda + 2) = 0 \quad \text{für } \lambda = 2 \text{ oder } \lambda = 1$$

$\lambda = 1$ $\quad \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 3 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -1 \\ 3 & 3 & -2 \end{pmatrix} \begin{matrix} \\ -2z_1 \\ -3z_1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 1 \\ 0 & -3 & 1 \end{pmatrix} \begin{matrix} \\ \\ -z_2 \end{matrix}$

$$\longrightarrow \begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} x_1 + 2x_2 - x_3 &= 0 \\ -3x_2 + x_3 &= 0 \end{aligned} \qquad \begin{aligned} x_1 &= \tfrac{1}{3}t \\ x_2 &= \tfrac{1}{3}t \end{aligned}$$

$$x_3 = t$$

$$\vec{x} = \begin{pmatrix} \frac{1}{3}t \\ \frac{1}{3}t \\ t \end{pmatrix} = \frac{1}{3}t \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = t' \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$\lambda = 2$ $\quad \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 3 & 3 & -3 \end{pmatrix} \begin{matrix} \\ -z_1 \\ -3z_1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad x_1 + x_2 - x_3 = 0$

$$x_2 = t \qquad x_3 = s \qquad x_1 = -t + s$$

$$\vec{x} = \begin{pmatrix} -t+s \\ t \\ s \end{pmatrix} = \begin{pmatrix} -t \\ t \\ 0 \end{pmatrix} + \begin{pmatrix} s \\ 0 \\ s \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$